1.3 Linear Equations and Linear Inequalities MATH 161 THOMPSON

1) $\frac{1}{4}x = \frac{3}{20}$ Multiply all by 20 5x = 3 x = $\frac{3}{5}$

2) 3t – 4 = 24 – t 4t = 28 x = 7

3) 1 + 6n = 9n + 4 -3n = 3 x = -1

4) 3x – (4x+9) = 2x - 12 3x – 4x – 9 = 2x – 12

-x – 9 = = 2x – 12

-3x = -3 x = 1

5) An equation of the form ax + b = 0 is called a linear equation or a first degree.

6) $\left(\frac{5}{3}x + 4 = \frac{1}{3} + \frac{1}{3}x\right)$ Multiply all by 3 5x + 12 = 1 + x

4x = -11 x = $\frac{-11}{4}$

7) $\left(\frac{2}{3}x - 3 = \frac{5}{6}x\right)$ Multiply all by 6 4x - 18 = 5x

-x = 18 x = -18

8) $\left(\frac{3}{4}p = \frac{4}{5}p + \frac{3}{2}\right)$ Multiply all by 20

15p = 16p + 30

-p = 30 p = - 30

10 10 10

9) $(0.4t = 0.3 + 0.5t)$ Multiply all by 10 $4t = 3 + 5t$

$-t = 3$ $t = -3$

28 28 28

10) $\left(\frac{x+1}{4} + \frac{x+2}{7} = 1\right)$ Multiply all by 28 $7(x + 1) + 4(x + 2) = 28$

$7x + 7 + 4x + 2 = 28$

$11x + 15 = 28$ $11x = 13$ $x = \frac{13}{11}$

11) $(x+10)(x-4)=(x+2)^2$ **FOIL** left side and square the right side

$x^2 - 4x + 10x - 40 = x^2 + \mathbf{4x} + 4$

$6x - 40 = 4x + 4$

$2x = 44$ $x = 22$

12) $x(2x-3)=(2x+4)(x-7)$ **DISTRIBUTE** left sides out **FOIL** right side

$2x^2 - 3x = 2x^2 - 14x + 4x - 28$

$-3x = -10x - 28$

$7x = -28$ $x = -4$

13) $z(z^4 + 1) = 2 + z^5$ **DISTRIBUTE** left side

$z^5 + z = 2 + z^5$

$z = 2$

14) $ax - b = c$ solve for x move b to the right $ax = b + c$

divide by a $x = \frac{b+c}{a}$

15) $\frac{1}{p}+\frac{1}{s}=\frac{1}{b}$ solve for b Multiply all by psb

sb + pb = sp

b(s + p) = sp using the two other variables s and p

b = $\frac{sp}{s+p}$ ** *multiply on top and add on bottom*

Another ex

ab ab ab

15) $\left(\frac{2x}{b}-\frac{x}{a}=c\right) solve\ for\ x$ Multiply all by **ab**

2xa – xb = abc Factor out the x x(2a-b) = abc

divide to get x by itself x = $\frac{abc}{2a-b}$

16) Todd is paid time-and-a-half over 40 hours. If he grosses $345 for 44 hours, what it his regular hourly rate? Overtime hours: 44 – 40 = 44

Let x = hourly rate 40x + 4(1.5x) = 345

40x + 6x = 345 x = $7.50

17) Going into a final exam which will count as two tests, Shawn has test scores 79, 80, 70, 65, and 96. What score does he need to make on the final to average and 80? *The average is the sum divided by the number of items (7 scores)*

$\frac{79+80+70+65+96+2x}{7}=80$

2x + 390 = 560 2x = 170 x = 85

18) A builder reduced the price of a house by 15%. If the new price is $595,000, what was the original price? * divide $ by (1-%) *1 - 0.15=0.85*

x = $\frac{595000}{0.85}$ x = $700,000

Reduced ----- Divide by 1-

19) A college bookstore marks up the book price by 25%. If they sell that book for $81.00, what did they pay for the book? * divide $ by (1+%) *1 + 0.25=1.25*

$$x = \frac{81.00}{1.25} \quad x = \$64.80$$

Marks up ----- Divide by 1+

20)) The perimeter of a window is 42 feet. The width is 5 feet more than the length, what are the dimensions?

P=2ℓ + 2w w = 5 + ℓ

ℓ is length and w is width

42 = 2ℓ + 2(5 + ℓ)

42 = 2ℓ + 10 + **2**ℓ

42 = 4ℓ + 10

32 = 4ℓ ℓ = **8**

w = 5+8 = **13**

< > means ( ) ≤ ≥ means [ ]

right arrow ∞) left arrow (- ∞

21) Express the graph shown in color using interval notation. Also express it as an inequality involving x.

Use interval notation to describe the graph. What is the resulting interval?

Interval Notation [2,6] Inequality Notation $2 \leq x \leq 6$

Means between, inequality is always ≤

22) Express the graph shown in color using interval notation. Also express the graph as an inequality involving x.

Interval Notation [-1,∞)

Inequality Notation $x \geq -1$ infinity on the right because > points right

23) $-2 \leq x < 8$ Interval notation [-2,8)

Means between: ≤ is a bracket < is parenthesis

24) $x > 1$ Interval notation (1,∞)

infinity on the right because > points right

25) $\frac{1}{6}(x - 15) < x + 20$ **multiply the right side by 6** to eliminate fraction

$x - 15 < 6x + 120$

$-5x < 135$ *switch inequality when dividing by negative*

$\{x|x > -27\}$ or $(-27,\infty)$

infinity on the right because > points right

26) $0 \leq 2x - 8 \leq 6$ Solve the middle for x add 8 to all 3 parts

$8 \leq 2x \leq 14$ divide all parts by 2

Inequality notation $\{x|4 \leq x \leq 7\}$ Interval Notation [4,7]

Means between

Extra Examples

a) $\frac{x}{x-2}+3=\frac{2}{x-2}$ Multiply all by (x-2) x + 3x-6 = 2

4x = 8 x = 2; however, that will make the denominator zero and that is undefined so the answer is NO SOLUTION

b) $\frac{2x}{x^2-4}=\frac{4}{x^2-4}-\frac{2}{x+2}$ Multiply all by (x^2-4) same as (x-2)(x+2)

2x = 4 – 2(x-2)

2x = 4 – 2x + 4

4x = 8; however that will make the denominator zero and that is undefined so the answer is NO SOLUTION

c) $\frac{x}{x+4}=\frac{7}{6}$ Cross multiply to get 7x + 28 = 6x

x = -28

d) $\frac{6t+7}{4t-7}=\frac{3t+7}{2t-5}$ Cross multiply (6t+7)(2t-5) = (4t-7)(3t+7)

$12t^2$ -16t-35 = $12t^2$ +7t-49

-23t = -14 x = $\frac{14}{23}$

e) $\frac{5}{x-3}=\frac{-4}{x+5}+\frac{31}{(x-3)(x+5)}$ Multiply all by (x-3)(x+5)

5(x+5) = -4(x-3) + 31

5x+25= -4x +43 9x = 18 x = 2

f) $\frac{x}{x^2-1}-\frac{x+3}{x^2-x}=\frac{-3}{x^2+x}$ Factor each denominator

(x-1)(x+1) x(x-1) x(x+1)

Multiply all by x(x-1)(x+1)

x(x) – (x+3)(x+1)= -3(x-1) *distribute the negative in the foil part*

$x^2 – x^2 – 4x – 3 = -3x +3$

– 4x – 3 = -3x +3

-x = 6 x = -6

g) $\frac{1}{s}+\frac{1}{p}=\frac{1}{g}\ solve\ for\ g$ Multiply all by **spg** pg + sg = sp

Factor out the g g(p+s) = ps divide to get g by itself g = $\frac{ps}{p+s}$

h) $D=\frac{pz^3}{T}\ solve\ for\ T$ Multiply both by **T** $DT=pz^3$

Divide by D $T=\frac{pz^3}{D}$

i) $(8x+4)^{-1}<0$ Negative exponent makes a fraction

$\frac{1}{8x+4}<0$ 8x+4 < 0 then x < -½

Inequality notation x < -½ Interval Notation (-∞,-½)

j) $(8x+4)^2<0$ Square can never be negative therefore NO SOLUTION

k) $(8x+4)^{-2}>0$ Square will always be positive therefore ALL REALS

l) Domain of $\sqrt{6x+30}$ Cannot have negative under the radical

6x + 30 ≥ 0

x ≥ -5

m) $\frac{1}{2}x+\frac{1}{3}x-\frac{1}{5}(x+6)\leq\frac{1}{10}$ multiply all by 30 (common denominator)

$15x+10x-6(x+6)\leq 3$ $19x\leq 39$ x ≤ $\frac{39}{19}$

Interval Notation (-∞,$\frac{39}{19}$) Inequality notation x ≤ $\frac{39}{19}$

n) -2(3x-4) > 20 Solve for x as a normal equation

-6x + 8 > 20

$-6x > 12$ *switch inequality when dividing by negative*

$x < -2$

Inequality notation $\{x \mid x < -2\}$ Interval Notation $(-\infty, -2)$